TRINITY COLLEGE

Year 11 Mathematics Specialist 1,2
Test 4   2021

Section 1   Calculator Free
Trigonometry

STUDENT'S NAME   _____

DATE: Friday 30 July   **TIME:** 35 minutes   **MARKS**: 39

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1.   (5 marks)

Given $\tan \vartheta = \dfrac{3}{7}$ and $180° \le \vartheta \le 270°$, determine

(a)   the exact value of $\sin \vartheta$ [2]

(b)   the exact value of $\cos \dfrac{\vartheta}{2}$ [3]

2.   (10 marks)

(a)   Express $\sqrt{3}\cos x + \sin x$ in the form $R\sin(x+\alpha)$ for α, an acute angle in radians.   [4]

(b)   Determine the minimum value of the expression in (a) and determine the smallest positive value of x for which this occurs.   [3]

(c)   Hence or otherwise, solve the equation $\sqrt{3}\cos x + \sin x = 1$ for $0 \le x \le 2\pi$.   [4]

3.   (8 marks)

(a)   Prove $\dfrac{1-\cos 2\vartheta + \sin 2\vartheta}{1+\cos 2\vartheta + \sin 2\vartheta} = \tan \vartheta$ [4]

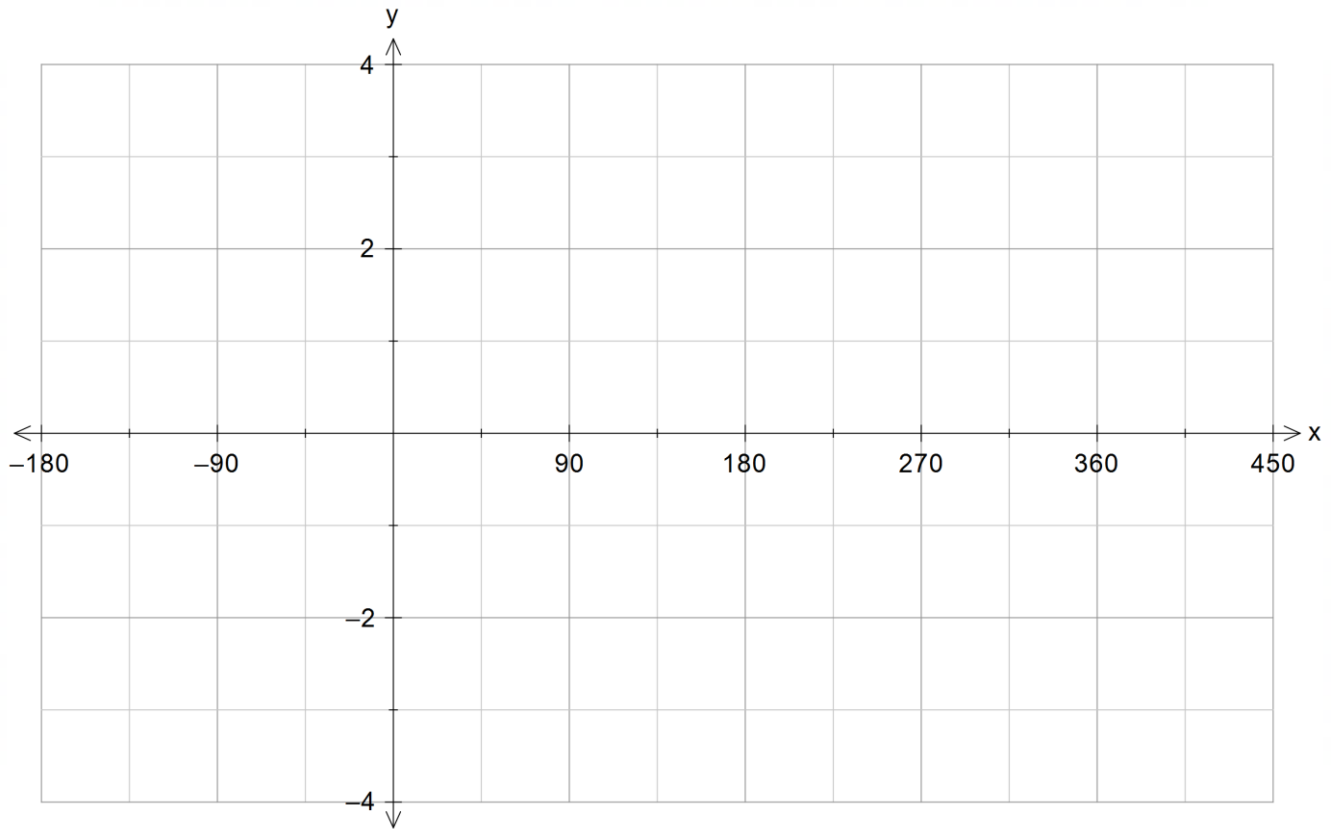(b)   Using the result of (a), show $\tan 15° = 2 - \sqrt{3}$ [4]

4.   (5 marks)

If $(\sin A + \cos B)^2 + (\cos A + \sin B)^2 = 3$, determine two possible values for the angle $(A+B)$ where $0 \le (A+B) \le 2\pi$.

5.   (6 marks)

Solve $0.5\sec(2\vartheta - \dfrac{\pi}{3}) = 1$

6. (4 marks)

Sketch the graph of $y = -2\operatorname{cosec}(x - 45°)$ on the axes below.

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Year 11 Mathematics Specialist 1,2
Test 4   2021

Section 2   Calculator Assumed
Trigonometry

STUDENT'S NAME _____

DATE: Friday 30 July   **TIME:** 15 minutes   **MARKS**: 17

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)
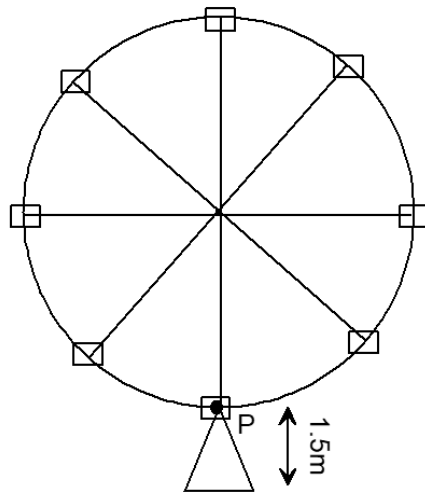
Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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7.   (5 marks)

Prove $\dfrac{1+\cot\alpha}{\operatorname{cosec}\alpha} - \dfrac{\sec\alpha}{\cot\alpha + \tan\alpha} = \cos\alpha$

8. (12 marks)

The height above ground of a person sitting in a cart on a Ferris Wheel can be modelled by a trigonometric function.


(a) Paulo is sitting on a chair in a Ferris Wheel of radius 8 m. His starting position is 1.5 m above ground as shown in the diagram above. The Ferris Wheel moves around anticlockwise, at a constant velocity, one revolution every 24 seconds.

(i) Draw a graph representing how Paulo's height changes over time each [3] revolution, given t is in seconds and y is Paulo's height in metres.
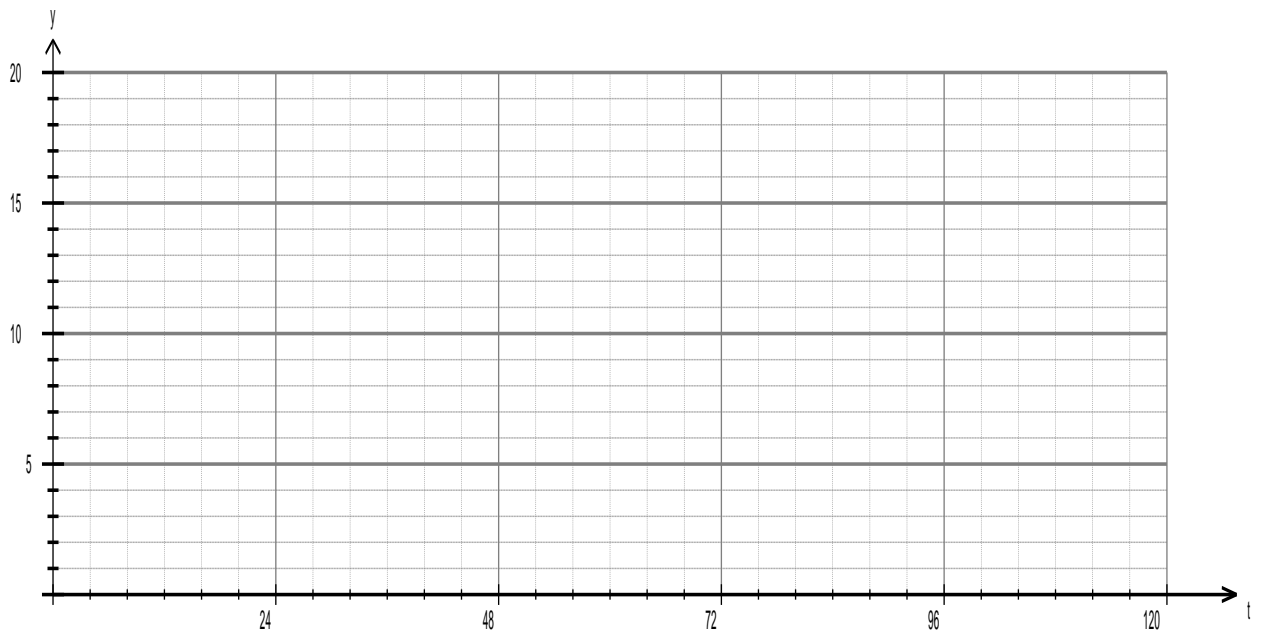
(ii) Determine the maximum height reached by Paulo and the times this occurs if the Ferris Wheel stops after 2 minutes. [3]

(b)   The graph below shows Matia's height above ground over time when sitting in a cart of a different Ferris Wheel to Paulo. This Ferris Wheel also moves around anticlockwise.
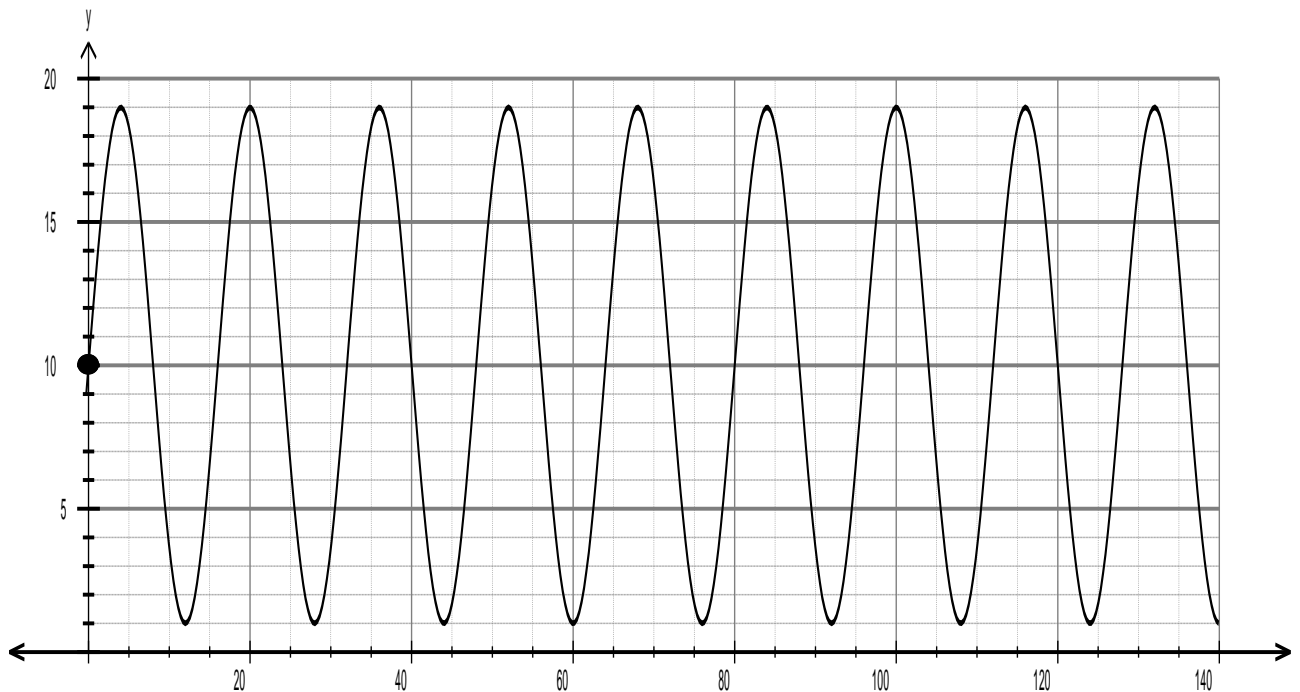
Matia's height above ground over time is modelled by the equation
    $y = a \sin bt + c$
    where y is Matia's height above the ground in m, at time t, secs and
    a, b and c are constants.

(i)   What is the radius of this Ferris Wheel?   [1]

(ii)   Determine the time taken for one revolution.   [1]

(iii)   Determine the value of a, b and c.   [3]

(iv)   On the Ferris Wheel below, indicate Matia's starting position.   [1]